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THE APPLICABILITY OF A HEREDITARY MODEL OF WEAR WITH AN EXPONENTIAL KERNEL IN THE ONE-DIMENSIONAL CONTACT PROBLEM TAKING FRICTIONAL HEAT GENERATION INTO ACCOUNT†

A. A. YEVTUSHENKO and Yu. A. PYR'YEV

Lvov

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An analytic solution is obtained for the contact problem for a stiff thermally insulated plate and an elastic heat-conducting layer, subject to the conditions of wear and frictional heating, when the contacting bodies are not drawn nearer. The evolution of the contact pressure, the temperature and the wear are traced. Conditions for the occurrence of thermoelastic instability are established. The conditions under which the wear model considered is applicable are given. © 2000 Elsevier Science Ltd. All rights reserved.

1. INTRODUCTION

As a rule, the models of wear used in contact problems assume a finite algebraic dependence of the rate of wear on the pressure P, the rate of slip V and other parameters [1–3]. However, under variable loading conditions in the tribosystem, relaxation effects may also be observed [4]. In general, the wear relationship is then a functional of the hereditary type, the form of which can be found experimentally. The hereditary model takes account of the relaxation of the contribution to the overall wear over a time t from all of the preceding perturbations at instants of time τ using a memory function $K(t-\tau)$. In particular, a wear relationship with an exponential kernel

$$U^w(t) = K_U V \int_0^t P(\tau) K(t-\tau) d\tau, \quad K(t-\tau) = e^{-\dot{\gamma}'(t-\tau)} \tag{1.1}$$

has been used in [5] which describes, as was noted in [6], processes with a restricted amount of wear. It is characteristic in the case of this relationship that the rate of wear

$$\partial U^w(t)/\partial t = -\dot{\gamma} U^w(t) + K_U V P(t)$$

will not be "*a priori*" positive for all values of the parameter γ' and the wear coefficient K_U.

The temperature and contact pressure fields and the field for the amount of wear of an elastic heat-conducting layer which is compressed by a stiff, uniformly sliding, thermally insulated plate are investigated below under conditions of frictional heating. It was found that an hereditary wear model of the form of (1.1) can be used for any relations between the input parameters of the problem. The conditions for the occurrence of frictional thermoelastic instability (FTEI) of the system are also found. By the latter term, we mean an unbounded increase in the contact characteristics (temperature, pressure and wear) when there is a small change in the input parameters.

Thermoelastic instability has been investigated in structures of the type of end and radial gaskets when there are axially symmetric perturbations [7]. The conditions for the occurrence of a thermal explosion (an unlimited contact temperature) and thermal power stability (a decrease in the pressure and temperature at the contact with time) have been considered in the problem of the contact interaction of an elastic layer with a stiff punch [8] and in the case of the non-ideal thermal contact of two compressible uniformly sliding layers [9], under the assumption that the temperature dependence of the friction coefficients and wear resistance is linear; the heat conduction in the layers was assumed to be quasisteady. The problem of the stability of the quasisteady-state solution of the thermoelastic wear problem has also been considered [10].

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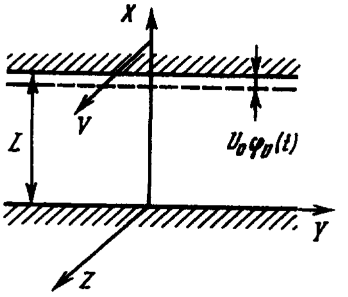

Fig. 1.

2. FORMULATION AND SOLUTION OF THE PROBLEM

Suppose an elastic heat conducting layer of thickness (ν is the Poisson's ratio, E is Young's modulus, and λ, k and α are the heat transfer, sufficient thermal conductivity and coefficient of linear expansion, respectively) is rigidly fastened along its base and compressed on its upper face by an amount $U = U_0\varphi_U(t)$ by means of a rigid thermally insulated infinite plate (Fig. 1). The plate slides in the direction of the Z axis at a constant velocity V. Due to friction forces, heat will be evolved and wear will occur in the contact region between the plate and the layer, and heat transfer will occur in accordance with Newton's law between the base of the layer and the surrounding medium.

It is required to determine the temperature increment field $T(X, t)$, the thermoelastic displacements $U(X, t)$, $W(X, t)$ along the X and Z axes, respectively, and the amount of wear of the layer $U^w(t)$.

The problem can be reduced to solving the system of differential equations of Lamé–Neumann quasistatic uncoupled thermo-elasticity [11]

$$\frac{\partial^2}{\partial X^2}U(X,t) = \alpha\frac{1+\nu}{1-\nu}\frac{\partial}{\partial X}T(X,t),\ \frac{\partial^2}{\partial X^2}W(X,t) = 0$$

$$\frac{\partial^2}{\partial X^2}T(X,t) = k^{-1}\frac{\partial}{\partial t}T(X,t),\ 0 < X < L,\ 0 < t < t_c \tag{2.1}$$

with the mechanical conditions

$$U(0,t) = W(0,t) = 0,\ \sigma_{XZ}(L,t) = fP(t)$$

$$U(L,t) = -U_0\varphi_U(t) + U^w(t),\ 0 < t < t_c \tag{2.2}$$

the thermal conditions

$$\lambda\frac{\partial}{\partial X}T(0,t) = hT(0,t)$$

$$\lambda\frac{\partial}{\partial X}T(L,t) = fVP(t),\ 0 < t < t_c \tag{2.3}$$

and the initial conditions

$$T(X, 0) = 0,\ 0 < X < L \tag{2.4}$$

Here, f is the friction coefficient, h is the heat transfer coefficient, and the contact time t_c is defined as the time during which the contact pressure $P(t) = -\sigma_{XX}(L, t)$ is non-negative ($P(t) \geqslant 0$ when $t \in (0, t_c]$).

The normal and shear stresses for the layer are found using the Duhamel–Neumann relations [11]

$$\sigma_{XX} = \frac{E}{1-2\nu}\left[\frac{1-\nu}{1+\nu}\frac{\partial U}{\partial X} - \alpha T\right],\ \sigma_{XZ} = \frac{E}{1+\nu}\frac{\partial W}{\partial X} \tag{2.5}$$

Taking relations (2.5) and boundary conditions (2.2) into account, we find from Eq. (2.1) that

$$P(t)=\frac{E_1}{L}\left[U_0\varphi_U(t)-U^w(t)+\alpha_1\int_0^L T(\xi,t)d\xi\right]$$
$$W(X,t)=\frac{1+\nu}{E}fP(t)X \quad (2.6)$$
$$E_1=\frac{E(1-\nu)}{(1+\nu)(1-2\nu)},\quad \alpha_1=\alpha\frac{1+\nu}{1-\nu}$$

We now introduce the dimensionless quantities

$$x=\frac{X}{L},\quad \tau=\frac{t}{t_*},\quad \tau_c=\frac{t_c}{t_*},\quad u=\frac{U}{U_0},\quad v=\frac{Vf\Omega}{V_*},\quad u^w=\frac{U^w}{U_0}$$
$$\theta=\frac{T}{T_*},\quad p=\frac{P}{P_*},\quad \Omega=\frac{E\alpha k}{\lambda(1-2\nu)},\quad \xi=\frac{E_1K_U}{f\Omega},\quad \gamma=\gamma' t_*,\ \mathrm{Bi}=\frac{hL}{\lambda}$$

and the characteristic parameters

$$t_*=\frac{L^2}{k},\quad V_*=\frac{k}{L},\quad T_*=\frac{U_0}{\alpha_1 L},\quad P_*=\frac{E_1U_0}{L}$$

We then obtain the heat conduction boundary-value problem

$$\frac{\partial^2}{\partial x^2}\theta(x,\tau)=\frac{\partial}{\partial\tau}\theta(x,\tau),\quad 0<x<1,\quad 0<\tau<\tau_c$$
$$\frac{\partial}{\partial x}\theta(0,\tau)=\mathrm{Bi}\,\theta(0,\tau) \quad (2.7)$$
$$\frac{\partial}{\partial x}\theta(1,\tau)=v\,p(\tau),\quad 0<\tau<\tau_c$$
$$\theta(x,0)=0,\ 0<x<1$$

where the dimensionless pressure and wear are respectively equal to

$$p(\tau)=\varphi_U(\tau)-u^w(\tau)+\int_0^1\theta(\xi,t)d\xi \quad (2.8)$$
$$u^w(\tau)=\xi v\int_0^\tau p(\eta)K(\tau-\eta)d\eta$$

We will obtain the solution of boundary-value problem (2.7), (2.8) using a Laplace integral transformation with respect to the time τ where, in order to transfer into the domain of the originals, we make use of the theorems for the expansion and multiplication of transforms [12]. As a result, we obtain the solution in the form

$$\theta(x,\tau)=v\varphi_U(\tau)_*\frac{d}{d\tau}g(x,\tau)$$
$$p(\tau)=\varphi_U(\tau)+v\varphi_U(\tau)_*\frac{d}{d\tau}G(\tau) \quad (2.9)$$
$$u^w(\tau)=v\,\varphi_U(\tau)_*\frac{d}{d\tau}I(\tau)$$

where

$$g(x,\tau)=g_1(x)+\sum_{m=1}^{\infty}\frac{\Delta_3(x,s_m)}{s_m\Delta'(s_m)}e^{s_m\tau}$$

$$G(\tau)=G_0+\sum_{m=1}^{\infty}\frac{\Delta_4(s_m)}{s_m\Delta'(s_m)}e^{s_m\tau}\quad I(\tau)=I_1+\sum_{m=1}^{\infty}\frac{\xi\Delta_1(s_m)}{s_m\Delta'(s_m)}e^{s_m\tau} \tag{2.10}$$

$$g_1(x)=\frac{G_0\,\mathrm{Bi}_x\,\xi_c}{\xi_c-\xi},\ G_0=\frac{1}{\upsilon_1-\upsilon},\ I_1=\frac{\xi}{\xi_c-\xi}G_0,\ \xi_c=\frac{\gamma}{\upsilon_0}$$

$$\upsilon_1=\frac{\upsilon_0\xi_c}{\xi_c-\xi},\ \upsilon_0=\frac{\mathrm{Bi}}{1+\mathrm{Bi}/2},\ \mathrm{Bi}_x=\frac{1+x\,\mathrm{Bi}}{1+\mathrm{Bi}/2}$$

$$\Delta_1(s_m)=\mathrm{Bi}\,C_m+s_mS_m,\ \Delta_2(s_m)=S_m-\mathrm{Bi}\,C_m^0$$

$$\Delta_3(x,s_m)=(s_m+\gamma)(\mathrm{Bi}\,S_m^x+C_m^x),\ C_m^0=-(C_m-1)/s_m$$

$$\Delta_4(s_m)=(s_m+\gamma)\Delta_2(s_m)-\xi\Delta_1(s_m)$$

$$\Delta'(s_m)=\frac{1}{2}\{(s_m+\gamma)[(\mathrm{Bi}+1)S_m+C_m-\upsilon s_m^{-1}[C_m-S_m+\mathrm{Bi}(S_m+2C_m^0)]]+$$

$$+2[\Delta_1(s_m)-\upsilon\Delta_2(s_m)]+\xi\upsilon\,[(1+\mathrm{Bi})S_m+C_m]\}$$

$$S_m=D_m^{-1}\,\mathrm{sh}\,D_m,\ S_m^x=D_m^{-1}\,\mathrm{sh}(D_mx),\ C_m=\mathrm{ch}\,D_m,\ C_m^x=\mathrm{ch}(D_mx),\ D_m=\sqrt{s_m}$$

An asterisk denotes convolution of functions with respect to time and s_m are the roots of the characteristic equation $\Delta(s)=0$ ($m=1,2,\ldots$).

We will now analyse these roots. $\mathrm{Im}\,s_m=0$, $\mathrm{Re}\,s_m<0$ when $m=3,4,\ldots$, and, when $m=1,2$, the roots, depending on the initial parameters of the problem, are located in the right or left half of the complex plane s. When $\upsilon=\upsilon_{2,3}$, the roots s_1 and s_2 are identical and, when $\upsilon=\upsilon_b$, the real part of the complex conjugate roots s_1 and s_2 is equal to zero. When $\upsilon=\upsilon_1$, one of the roots is equal to zero.

The case when $\gamma<\gamma_*$. When $\xi<\xi_*$, where

$$\gamma_*=\frac{\upsilon_0}{\mathrm{Bi}_2},\ \mathrm{Bi}_2=\frac{1+5\mathrm{Bi}/6+5\mathrm{Bi}^2/24}{(1+\mathrm{Bi}/2)^2},\ \xi_*=\xi_c\frac{\gamma}{\gamma_*}$$

and $\upsilon<\upsilon_2$ and $\upsilon_3<\upsilon<\upsilon_1$, the roots are negative, when $\upsilon_2<\upsilon<\upsilon_3$, they are complex conjugate with a negative real part and when $\upsilon_1<\upsilon$ the root s_1 is positive and the root s_2 is negative. When $\xi_*<\xi<\xi_c$, $\upsilon<\upsilon_b$, the roots are complex conjugate with a negative real part, when $\upsilon_b<\upsilon<\upsilon_3$ they are complex conjugate roots with a positive real part and when $\upsilon>\upsilon_3$ the roots are positive. However, if, additionally, $\upsilon>\upsilon_1$, then the root s_2 becomes negative. When $\xi_c<\xi<\xi_1$, $\upsilon<\upsilon_b$ the roots are

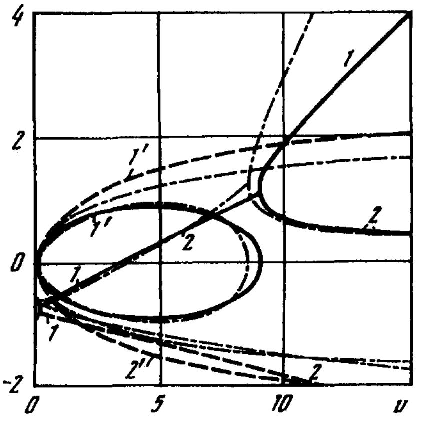

Fig. 2.

complex conjugate with a negative real part, when $\upsilon_b < \upsilon < \upsilon_3$ they are complex conjugate roots with a positive real part and when $\upsilon_3 < \upsilon$ the roots are positive. When $\xi_b < \xi$ and $\max(\upsilon_2, 0) < \upsilon < \upsilon_3$ if $\upsilon_3 > 0$, or $\max(\upsilon_2, 0) < \upsilon < \infty$, if $\upsilon_3 < 0$, the roots are complex conjugate with a negative real part, and they are negative when $0 < \upsilon_3 < \upsilon$.

The change in the real and the negative parts of the roots s_1 and s_2 as a function of the dimensionless velocity υ is shown in Fig. 2 for the values $\gamma = 0.5$, Bi = 1.43. The roots *1* and *1'* (2 and 2') correspond to the real and imaginary parts of the root s_1 (s_2). The solid and dashed lines refer to the values $\xi = 0.7$ and $\xi = 1.4$, respectively.

The case when $\gamma > \gamma_*$. When $\xi < \xi_c$ the roots are negative if $\upsilon < \upsilon_1$. If $\upsilon > \upsilon_1$ the root s_1 is positive and the root s_2 is negative. When $\xi_c < \xi$ the roots are negative for an arbitrary velocity υ.

In the neighbourhood of zero, the roots s_1 and s_2 can be written in the explicit form

$$s_{1,2} = -\tilde{\alpha} \pm \tilde{\omega} \tag{2.11}$$

$$\tilde{\alpha} = b/(2a), \quad \tilde{\omega} = |\tilde{\alpha}|\sqrt{1-4ac/b^2}$$

$$a = a_2(v_a - v), \quad b = b_2(v_b - v), \quad c = c_2(v_1 - v), \quad a_2 = (\xi_a - \xi)\mathrm{Bi}_4$$

$$b_2 = \xi_b - \xi, \quad c_2 = (\xi_c - \xi)v_0, \quad \xi_b = 1 + \gamma \mathrm{Bi}_4, \quad \xi_a = 1 + \gamma \mathrm{Bi}_3/\mathrm{Bi}_4$$

$$v_a = \frac{\xi_b}{a_2}, \quad v_b = \frac{\gamma + v_0}{b_2}, \quad \mathrm{Bi}_3 = \frac{1+\mathrm{Bi}/6}{120(1+\mathrm{Bi}/2)}, \quad \mathrm{Bi}_4 = \frac{1+\mathrm{Bi}/4}{6(1+\mathrm{Bi}/2)}$$

The change in the real and imaginary parts of the approximation of the roots s_1 and s_2 using formulae (2.11) is shown as a function of the dimensionless velocity υ for $\xi = 0.7$ and $\xi = 1.4$ by the dot–dash curves in Fig. 2.

Relations (2.11) enable us to write approximate expressions for the values of υ_m, $m = 2, 3$ and ξ_1.

3. THEORETICAL ANALYSIS OF THE RESULTS

As an example, we will investigate the characteristics of thermoelastic contact in a wear process at a value of the compression of the layer which is constant with respect to time, that is $\varphi_U(\tau) = H(\tau)$, where $H(\cdot)$ is the Heaviside function.

The following asymptotic forms of the temperature, pressure and wear at sort time were obtained by an analytic investigation of the properties of solutions (2.9)

$$\theta(1,\tau) = 2v\sqrt{\tau/\pi} + O(\tau^{3/2}), \quad p(\tau) = 1 + v(1-\xi)\tau + O(\tau^2)$$

$$u^w(\tau) = v\,\xi\tau + O(\tau^2); \quad \xi = \lambda K_U(1-\nu)/[f\alpha k(1+\nu)] \tag{3.1}$$

Relations (2.11)–(3.1) and the graphs in Fig. 2 enable us to predict the behaviour of the characteristics of the thermoelastic contact of the layer under conditions of frictional heating and wear.

The parameter ξ characterizes the mutual effect of the wear and thermal expansion.

When there is no wear ($\xi = 0$ or $\gamma = \infty$) at slip rates below the critical value υ_0 the contact pressure and temperature reach steady values

$$p_c = \frac{v_0}{v_0 - v}, \quad \theta_c(x) = \frac{x\mathrm{Bi}+1}{\mathrm{Bi}} p_c v$$

with time since, due to heat transfer, the inflow and outflow of heat in the system mutually compensate for one another. As the velocity υ approaches its critical value υ_0, the time required to reach steady-state conditions increases.

At velocities above the critical velocity ($\upsilon > \upsilon_0$), the temperature and contact pressure increase exponentially, that is, there is frictional thermoelastic instability (FTEI).

In the case of abrasive wear ($\gamma = 0$) or $0 < \xi < 1$ (when the thermal expansion predominates over the value of the wear), for $\upsilon \leqslant \upsilon_2$ the contact time $\tau_c = \infty$ and the contact characteristics tend with time to their own steady values $p_c = 0$, $\theta_c(x) = 0$, $u^w_c = 1$. The time required by the system to attain a steady state increases when $\upsilon \to \upsilon_2$. If $\upsilon_2 < \upsilon < c_3$, the contact time is limited. The minimum contact time will occur at velocities $\upsilon \approx (\upsilon_2 + \upsilon_3)/2$, that is, when the quantity $\operatorname{Im} s_1$ reaches its maximum value. As υ approaches υ_3, the maximum values of the pressure, temperature and wear increase. FTEI sets in at a velocity $\upsilon > \upsilon_3$.

When $\xi \geqslant 1$ (when the wear predominates over the value of the thermal expansion), for $\upsilon \leqslant \upsilon_2$ the characteristics of the contact tend with time to the steady values $p_c = 0$, $\theta_c(x) = 0$, $u_c^w = 1$. When $\upsilon \geqslant \upsilon_2$, the contact time τ_c is limited although a steady-state solution formally exists. Henceforth, "formally" means that, on reaching a steady-state solution, the function characterizing the pressure or the rate of wear changes sign. As the slip rate increases, the contact time decreases.

In the general case when $\gamma < \gamma_*$, $\xi < \xi_*$ and when $\upsilon < \upsilon_1$, the contact time $\tau_c = \infty$, and the contact characteristics tend, with time, to their own steady values

$$p_c = \frac{\upsilon_1}{\upsilon_1 - \upsilon}, \quad \theta_c(x) = \frac{x\mathrm{Bi}+1}{\mathrm{Bi}} p_c \upsilon, \quad u_c^w = \xi \upsilon \, p_c \tag{3.2}$$

The time taken to reach steady conditions increases as υ tends to υ_1. When $\upsilon > \upsilon_1$, there is FTEI. When $\xi_* < \xi < \xi_c$, $\upsilon < \upsilon_b$, the steady solution (3.2) formally exists (on reaching it, the rate of wear changes sign); when $u_b < \upsilon < \upsilon_3$, the contact time is limited and the rate of wear also changes sign. When $\upsilon > \upsilon_3$, there is FTEI. For $\xi_c < \xi < \xi_1$, $\upsilon < \upsilon_3$, the contact time is limited and the rate of wear is negative. FTEI is observed when $\upsilon > \upsilon_3$. When $\xi_b < \xi$, the steady solution (3.2) formally exists for all υ (the rate of wear also change sign).

Hence, when the system is not under conditions of FTEI an when $0 < \gamma < \gamma_*$, $\xi_* < \xi$, the wear function is not a monotonically increasing function and the use of the hereditary model (1.1) is inadmissible.

In the general case when $\gamma > \gamma_*$, $\xi < \xi_c$, $\upsilon < \upsilon_1$ the contact characteristics tend with time to the steady-state solution of problem (3.2). There is FTEI when $\upsilon > \upsilon_1$. When $\xi_c < \xi$, the contact pressure and temperature reach a steady state with time for all υ; in this case the contact pressure does not exceed the initial value.

When $\xi \geqslant 1$, with time the contact pressure always monotonically tends to zero (unlike the case when $0 < \xi < 1$ when it has a maximum).

4. NUMERICAL RESULTS

In order to illustrate the above theoretical analysis of the behaviour of the contact characteristics a numerical solution of the problem for a layer of steel was obtained ($\alpha = 14 \times 10^{-6}$ °C^{-1}, $K = 21$ W m^{-1}°C^{-1}, $k = 5.9 \times 10^{-6}$ m^2/s, $\nu = 0.3$, and $E = 190 \times 10^9$ Pa) when $L = 3 \times 10^{-2}$ m, Bi = 1.43, $U_0 = 10^{-6}$ m and $f = 10^{-2}$ for values of the dimensionless slip velocity υ. The values of the parameters for rendering quantities dimensionless are then as follows: $t_* = 153$ s, $V_* = 1.97 \times 10^{-4}$ m/s, $P_* = 8.53 \times 10^6$ Pa and $T_* = 1.28$ °C^{-1}. The critical value of the parameter $\gamma_* = 0.936$ corresponds to the value Bi = 1.43. The value of the parameter $\gamma = 1.2$ was taken from the condition for the use of the hereditary wear model (1.1) to be admissible. Then, $\xi_c = 1.44$. The case when the amount of wear predominates over thermal expansion ($\xi = 1.5$) is considered.

Graphs of the dimensionless amount of wear u^w and the dimensionless contact pressure p against the dimensionless time τ are shown in Figs 3 and 4 for various values of the dimensionless velocity. It is clear that there is intensive wear during the initial stage, which leads to a sharp fall off in the contact pressure. With time,

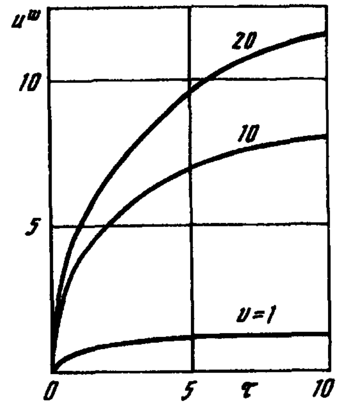

Fig. 3.

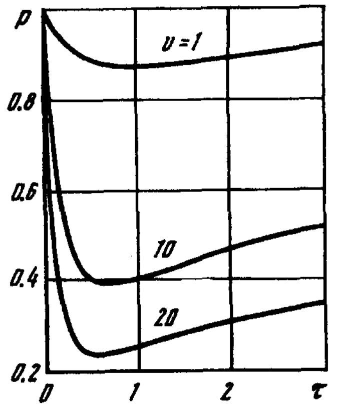

Fig. 4.

the rate of wear falls off as a consequence of running in, the pressure begins to increase and, together with the amount of wear, reaches the steady values (3.2).

5. CONCLUSIONS

An analytical solution has been obtained for the contact problem of a rigid thermally insulated plate and an elastic heat-conducting layer under conditions of wear and frictional heating in the layer when the contacting bodies are not drawn nearer. The evolution of the contact pressure, temperature and wear has been traced.

The determined analytical values for the characteristic values of the velocity υ_m, $m = 1, 2, 3$ and the parameters γ_*, ξ_c, ξ_*, ξ_1 enable one to predict the behaviour of the characteristics of the frictional contact with time.

The conditions for the occurrence of frictional thermoelastic instability (FTEI)

$$\gamma \in [0;\gamma_*], \quad \xi \in [0;\xi_*], \quad \nu \in [\nu_1;\infty)$$

$$\gamma \in [0;\gamma_*], \quad \xi \in [\xi_*;\xi_1), \quad \nu \in [\nu_3;\infty)$$

$$\gamma \in [\gamma_*;\infty), \quad \xi \in [0;\xi_c), \quad \nu \in [\nu_1;\infty)$$

have been established for the frictional contact model considered.

In the case of abrasive wear ($\gamma = 0$), the critical value of the velocity υ_3, at which FTEI occurs, increases, and when $\xi \geqslant 1$ (when wear predominates over thermoelastic expansion) FTEI completely disappears. Hence, abrasive wear emerges as a stabilizing factor.

Conditions have been obtained for the applicability of the hereditary wear model.

The generally accepted opinion that instability occurs when the zeros of the characteristic equation are located in the right half of the complex plane of the Laplace transform parameter needs to be revised in the case of this model of frictional contact. These roots must have a zero imaginary part. The existence of an imaginary part leads to boundedness of the contact time.

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